Lesson 1 - Derivative Review

Power Rule

Chain Rule $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Product Rule

Quotient Rule

$\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

$\frac{d}{dx}[\sin(x)] = \cos(x)$ $\quad$ $\frac{d}{dx}[\cos(x)] = -\sin(x)$

+ 4 other trig derivatives

$\frac{d}{dx}[c] = 0$

c ← constant

$\boxed{EX}$ $y = x(\cos(x) + 3e^x)$

Find $\left.\frac{dy}{dx}\right|_{x=0}$

$$\frac{dy}{dx} = 1(\cos(x) + 3e^x) + x(-\sin(x) + 3e^x)$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 1(\cos(0) + 3e^0) + \underbrace{0(-\sin(0) + 3e^0)}_{=0}$$

$$= 1(1 + 3(1))$$

$$= 4$$

$\boxed{EX}$ Find the derivative of $y = \frac{x^5 + 3\sqrt{x}}{x^{1.2}}$

$3x^{1/2}$

$$\frac{dy}{dx} = \frac{x^{1.2}(5x^4 + \frac{3}{2}x^{-1/2}) - (x^5 + 3\sqrt{x})1.2x^{0.2}}{(x^{1.2})^2}$$

$x^{2.4}$

Derivatives are Rates

position function (fctn) $\xrightarrow{\text{derivative}}$ velocity function

velocity function $\xrightarrow{\text{derivative}}$ acceleration function

Ex: A particle travels along a straight line. Its position at time t is given by

$$s(t) = 4(e^t + \sin(t))^3$$

Find the velocity of the particle at time t

$$v(t) = 12(e^t + \sin(t))^2 [e^t + \sin(t)]'$$
$$= 12(e^t + \sin(t))^2 (e^t + \cos(t))$$

Find the derivatives

quotient rule

(1) $y = \dfrac{15}{\sqrt[3]{x^2+1}} = \dfrac{15}{(x^2+1)^{1/3}} = 15(x^2+1)^{-1/3}$

$$\frac{dy}{dx} = -5(x^2+1)^{-4/3} \cdot 2x$$
$$= \frac{-10x}{(x^2+1)^{4/3}}$$

(2) $y = \sin(x^3)$

(3) $y = \sin^3(x)$

(2) $\dfrac{dy}{dx} = \cos(x^3) \cdot 3x^2$

(3) $y = (\sin(x))^3 \qquad \dfrac{dy}{dx} = 3(\sin(x))^2 \cdot \cos(x)$